How to choose a strategic multi-factor equity portfolio

James Barber, Chief Investment Officer – Equities
Scott Bennett, Director – Equity Strategy and Research
Evgenia Gvozdeva, Senior Quantitative Research Analyst

EXECUTIVE SUMMARY:

In this paper, we present two strategic multi-factor equity portfolios that combine four well-documented return sources: Value, Momentum, Quality, and Low Volatility to meet two distinct objectives. We propose a robust and flexible framework that uses the principles of modern portfolio theory and reduces the sensitivity to the estimation error.

Among other considerations, we believe investors can choose to focus on active risk return trade-offs or absolute risk return trade-offs. That is why, in this paper, we present two Strategic Portfolios: IR Portfolio (Information Ratio Portfolio) and SR Portfolio (Sharpe Ratio Portfolio). Their framework moves away from single-point estimates and captures the high-level relationships of return sources through a ranking approach. These high-level relationships have shown to hold through time and allow for consistent exposures through time. We also present the results for alternative weighting schemes such as equal-weight, minimum variance, maximum diversification, minimum correlation and risk parity.

Introduction

There has been growing interest in factor exposures as well as an increased number of publications advocating for the combination of different factor exposures in a single multi-strategy portfolio. The publications have included strategies ranging from as simple as equal-weighted allocations (see Bender [2014]) to maximum diversification strategies and even combinations of different weighting schemes (see Amenc et al. [2014]). While these publications have gone a long way in informing investors of the vast array of portfolio construction techniques available, they have provided little clarity as to which approach should be used and when. In this paper, we provide asset owners with a framework for multi-factor portfolio construction that is robust through time, designed to reduce estimation error, and utilises all the principles of modern portfolio theory.

The return of an equity portfolio can generally be decomposed into three components: strategic equity factor exposures, tactical equity factor tilting around the strategic policy, and specific security selection[1] (see Lo [2008]). In this paper, we focus on providing well-diversified exposures to the selected strategic return sources for building a strategic equity portfolio. The availability of the return streams of the factor portfolios allows us to do the analysis and choose the appropriate weighting schemes. It is also easier to express dynamic factor tilts away from the strategic portfolio weights when a multi-factor portfolio is constructed as a combination of factor portfolios. Short term forecasting is a difficult exercise but it should be easier to forecast factor portfolio returns than individual stock returns[2].

Combining multiple return sources in a strategic equity portfolio may help provide more consistent performance through time, just the same way as multiple return sources are combined in strategic asset allocation framework. When one factor is underperforming, another factor may be outperforming. A multi-factor portfolio acts to smooth different factor cycles and to reduce volatility associated with individual return sources. We demonstrate that the return sources we have chosen are lowly correlated in the active space and even provide some diversification benefits in the absolute space. The straightforward and flexible factor combination framework presented below adheres to the fundamental rules of modern portfolio theory but is more robust with respect to estimation error. Centrally managing the exposures to the return sources in an integrated portfolio helps reduce portfolio turnover, lower tax realisation, and also minimise transaction costs.

Return Sources

In this paper, we discuss four well-documented return sources: Value, Momentum, Quality, and Low Volatility[3]. When combining multiple factor returns, careful consideration needs to be given to the design and specification of the individual factor returns. We want to ensure that the factor strategies have common features that allow them to be combined without one unintentionally dominating the others. Using a consistent portfolio construction process across the factors results in similar levels of active share across the different factors and also similar factor exposure levels. For example, this would mean that the average active stock position in the Momentum portfolio is the same as the average active stock position in the Value portfolio. Further, the factor exposures in both portfolios should be of a similar magnitude. In this paper, we utilise the Russell Factor Exposure portfolios to represent the long-only factor return. The Russell Factor Exposure portfolios have many of the desired characteristics that we would look for in a factor portfolio, and they have shown to be robust proxies for the factors discussed in this paper. The portfolios are reconstituted semi-annually in June and December.

The Russell Factor Exposure portfolios are built using a methodology which we believe captures the cross section of the factor scores in the most efficient and sensible way. The non-linear weighting algorithm allows for a monotonic relationship between a stock's factor score and its active weight. This ensures that a higher factor score results in a higher exposure to a stock within the factor portfolio, but only marginally so at the extremes of the distribution. The relationship between factor characteristics and forward returns is often found to be non-linear (e.g., see Maslov and Rytchkov [2013]).

In Exhibit 1, we provide summary statistics for Russell Global Large Cap (LC) Factor Exposure portfolios, we use as proxies for the chosen return sources. All of the portfolios have shown outperformance against the Russell Global Large Cap Index over the period from July 1996 to July 2014. It is important to highlight here that we believe that Value, Momentum and Quality will outperform in the long run. Those beliefs are based on both economic intuition and longer historical perspective[4]. As expected, absolute volatility and semi-deviation of Low Volatility are lower than those of the benchmark. The maximum absolute drawdown over the studied period is the worst for Value. Low Volatility has the highest historical tracking error and maximum active drawdown, while Quality has the lowest historical tracking error and active drawdown.

---

[1] We define specific security selection here as alpha that is not explainable by strategic factor exposures or tactical factor tilting. It can include unrecognized factors and tilting around them.

[2] Forecasts for groups of items tend to be more accurate than forecasts for individual items. In the worst case, the forecasting error for the group is the sum of the individual forecasting errors.

[3] Although we don't include Size as an explicit factor, our factor portfolios are driven by this return source as well due to factor portfolio construction methodology and the nature of the factors.

[4] We believe that Low Volatility is an effective tool to control total volatility levels and to improve Sharpe ratio.

Exhibit 1: Russell Global LC Factor Exposure Portfolios Summary Statistics (Jul 1996 – Jul 2014, in USD)

	VALUE	VOLATILITY	MOMENTUM	QUALITY	RUSSELL GLOBAL LC
Annualised Return (CAGR)	9.9%	9.2%	9.4%	9.0%	7.5%
Annualised Volatility	17.2%	12.7%	17.4%	16.6%	16.4%
Sharpe Ratio	0.49	0.56	0.47	0.46	0.38
Semi-Deviation	12.7%	9.9%	13.6%	12.8%	12.6%
Sortino Ratio	0.67	0.73	0.60	0.59	0.49
Maximum Drawdown	-57.7%	-46.6%	-55.0%	-51.0%	-54.9%
Historical Beta	0.99	0.72	1.00	0.99	1.00
Excess Return (CAGR)	2.4%	1.7%	1.9%	1.5%	-
Tracking Error	5.4%	6.3%	5.4%	3.1%	-
T-stat of Excess Return	1.83	0.67	1.55	1.97	-
Information Ratio	0.43	0.16	0.36	0.46	-
Active Semi-Deviation	3.5%	4.4%	4.0%	2.1%	-
Active Sortino Ratio	0.66	0.23	0.48	0.68	-
Maximum Active Drawdown	-27.1%	-30.3%	-17.1%	-7.7%	-
Turnover	35%	25%	67%	29%	3%

Exhibit 2 shows the correlation matrix of absolute (total) returns. It is not a surprise that long only factors are highly correlated as they are all exposed to equity market beta, but with diversification potential. Value and Momentum returns are the least correlated series historically. Exhibit 3 shows that factors' excess returns are not highly correlated with each other and this presents an exploitable opportunity to minimise active risk. The cross-correlations vary by return sources, but the average cross-correlation across the four factors excess returns is close to zero.

Exhibit 2: Correlation of Russell Global LC Factor Exposure Portfolios Absolute Monthly Returns (Jul 1996 - Jul 2014, in USD)

	VALUE	LOW VOLATILITY	MOMENTUM	QUALITY
Value	1.00	-	-	-
Low Volatility	0.95	1.00	-	-
Momentum	0.86	0.87	1.00	-
Quality	0.91	0.89	0.96	1.00

Exhibit 3: Correlation of Russell Global LC Factor Exposure Portfolios Excess Monthly Returns (Jul 1996 - Jul 2014, in USD)

	VALUE	LOW VOLATILITY	MOMENTUM	QUALITY
Value	1.00	-	-	-
Low Volatility	0.43	1.00	-	-
Momentum	-0.44	-0.18	1.00	-
Quality	-0.33	-0.30	0.46	1.00

Although the factor excess returns are expected to be positive in the long run (5+ years), they are not consistently positive over shorter time horizons and changing regimes. Rolling one-year excess factor returns in Exhibit 4 demonstrate that performance cycles generally do not coincide. The Low Volatility factor demonstrates the strongest countercyclical pattern. At market bottoms, Value tends to start performing well due to a wide dispersion of valuation ratios across stocks and thus larger profit opportunities. At the same time, Momentum tends to start performing poorly because after a market crash it is overweight low beta stocks and underweight high beta stocks. At market tops, the opposite holds, as Value tends to start performing poorly while Momentum tends to start performing well. The Quality factor is the most stable in terms of returns, but still performs better during recessions when investors "fly to quality". This distinction in the sensitivities of the factors to different regimes makes them great candidates for a multi-factor portfolio with historically a more consistent performance pattern over time.

Exhibit 4: Russell Global LC Factor Exposure Portfolios 1-Year Rolling Excess Return (LHS) and Russell Global LC Total Return (RHS) (Jul 1997 – Jul 2014)
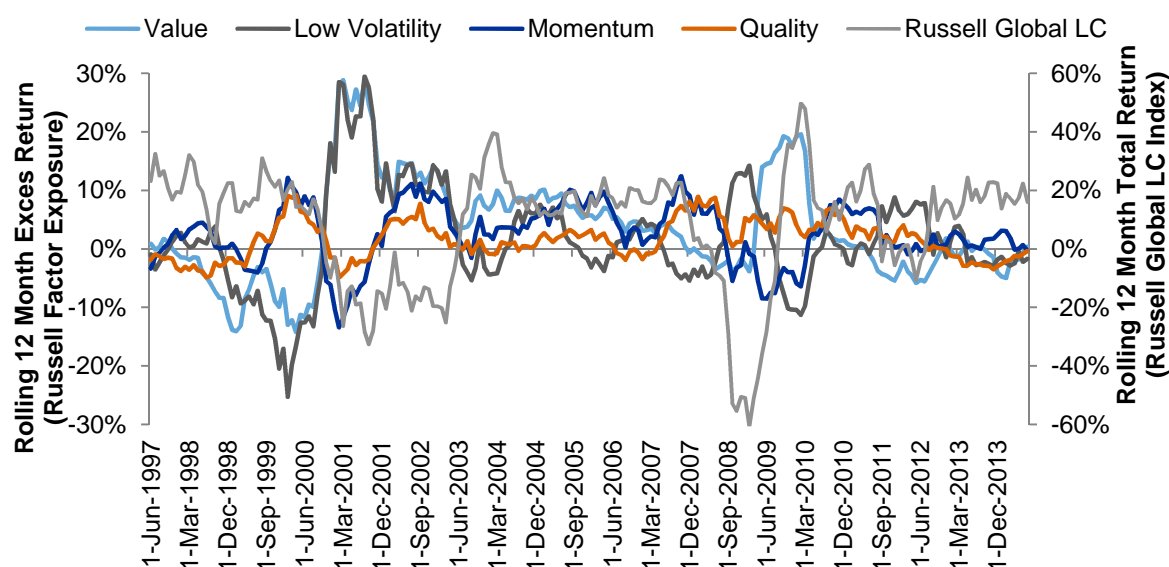
Multi-Factor Portfolio – Allocation Framework

We recognise that different asset owners have different objectives: some investors, such as investors in the accumulation phase, are focused on excess risk/return tradeoffs, and other investors, such as investors in the decumulation stage, are focused on absolute risk/return tradeoffs. Our portfolio construction framework is flexible and can easily accommodate both types of tradeoffs.

We have considered a number of other portfolio construction approaches (including minimum variance, maximum Sharpe ratio, maximum diversification, minimum correlation, risk parity and equal contribution to risk). We present the comparison in the next section. Here, we want to emphasise a robust non-parametric approach in determining the strategic weights to the selected return sources as our preferred method.

The estimation of expected returns and variance-covariance matrix is a difficult task. The realised parameters are often different from the estimates. A small error in the estimation of expected return can lead to a significant change in the optimal portfolio. To minimise the impact of estimation risk, we use ranks instead of the historical estimates of excess return expectations and the variance-covariance matrix. We rank across several regions (US, Developed ex US, and EM) to seek to improve robustness and to prevent a particular region alone from driving the factor allocation decision. Our simple and straightforward portfolio construction framework adheres to the fundamental rules of modern portfolio theory (about risk/return tradeoffs and correlations) while controlling for many of the well-documented issues found in optimised solutions (e.g., see Michaud [1989]). The

framework also delivers very intuitive outcomes[5]. We can exploit the flexibility of our approach even further by adding additional factors[6] and also by enhancing the ranking process (i.e., by adding additional characteristics that the investors might care about). Although we are not demonstrating it in this paper, a good example would be making a portfolio more tax friendly by adding another characteristic to the ranking process - tax exposure that comes from capital gains or dividend income for each factor portfolio. We present two strategic portfolios, which address preferences of the two groups of investors, respectively. The IR Portfolio is aimed towards those investors seeking a higher Information ratio (or active Sortino ratio) and the SR Portfolio is targeting those investors with a greater focus on a Sharpe ratio (or Sortino ratio).

First, we explain how we generate the allocations for the IR Portfolio which is focused on excess risk and return tradeoffs. The IR Portfolio takes into account the hierarchy of excess returns observed in history while penalising the return sources with least diversifying properties in the active space and highest active semi-deviations. For our framework, we focus on active semi-deviation as we acknowledge that downside risk is more relevant for the majority of investors.

In all the considered regions, we first rank the four return sources based on the historical excess returns, and then calculate the average rank across the regions for each return source. We then rank return sources the same way based on active semi-deviation. Finally, in order to rank the return sources by correlations, we calculate the average value for each row of the Pearson correlation matrix for each region (i.e., the average correlation of each factor with the three other factors in the region)[7]. We then rank the return sources by correlation with the lowest average correlation having the highest rank and the highest average correlation having the lowest rank. All the ranks are summarised in Exhibit 5.

Exhibit 5: Return Sources Average Ranks Across Regions and Final Ranks for IR Portfolio[8]

	VALUE	LOW VOLATILITY	MOMENTUM	QUALITY
Excess Return	1	3	3	4
Active Semi-deviation	2	4	3	1
Correlation of Excess Returns	2	4	1	3
Total	**1**	**4**	**2**	**3**

Value gets the highest overall rank, followed by Momentum, Quality, and Low Volatility; utilising the ranks we then convert these to portfolio weights by allocating higher weights to the better ranked return sources and lower weights to the lower ranked return sources, which translates into a fixed 40% Value/30% Momentum/20% Quality/10% Low Volatility heuristic weighting scheme for the IR Portfolio.

Next, we utilise the same framework to develop the SR Portfolio, which is focused on absolute risk and return tradeoffs. The SR Portfolio follows the same framework but takes into account the hierarchy of absolute returns observed in the history while penalising the return sources with least diversifying properties in the absolute space and highest absolute semi-deviations. All the ranks are summarised in Exhibit 6. Low Volatility gets the highest overall rank, followed by Value, Momentum, and Quality, which translates into a 40% Low Volatility/30% Value/20% Momentum/10% Quality heuristic weighting scheme for the SR Portfolio.

---

[5] We have also applied the proposed approach to the factor indexes from other providers and the conclusions have not changed.

[6] It has been recently shown that more than 330+ return sources were identified by academics since 1970 (see Greene et al. [2013]). In the end, though, only several equity factors likely matter the most and an investor needs to have the beliefs about which factors are important for her risk/return objectives and investment horizon.

[7] We use monthly data to estimate correlations and volatilities and we assume that the return distribution remains the same over the longer time period. Theoretically it would be better to use longer horizons (e.g. 60 month) to estimate correlations and volatilities in a strategic framework. Unfortunately, the period which we study yields only 3 independent observations for the 60-month correlation and volatility estimation. From a practical perspective, although the portfolio is strategic, its performance is monitored monthly and thus monthly correlations are important as well.

[8] To clarify, in each line, for Excess Return, Active Semi-deviation, and Correlation of Excess Returns, we show average rank across regions. E.g., for Excess Return, both Low Volatility and Momentum have an average rank of 3.

Exhibit 6: Return Sources Average Ranks Across Regions and Final Ranks for SR Portfolio

	VALUE	VOLATILITY	MOMENTUM	QUALITY
Absolute Return	1	3	3	4
Absolute Semi-deviation	3	1	4	2
Correlation of Absolute Returns	3	1	2	4
Total	**2**	**1**	**3**	**4**

General Characteristics of Strategic Portfolios relative to Russell Global LC Index

In Exhibit 7, we summarise the main characteristics of Global LC strategic portfolios. Historically, the IR Portfolio achieved a 34% improvement in the Sharpe ratio in comparison to Russell Global LC index. Interestingly, a simple equal-weighted portfolio is able to capture most of this improvement, with slightly more emphasis on volatility reduction than increasing returns. SR Portfolio achieved a 40% improvement in the Sharpe ratio in comparison to the benchmark.

Exhibit 7: Global LC Equity Strategic Portfolios Summary Statistics (Jul 1996 – Jul 2014)[9]

	STRATEGIC IR	STRATEGIC SR	EQUAL-WEIGHT	RUSSELL GLOBAL LC
Annualised Return (CAGR)	9.7%	9.6%	9.5%	7.5%
Annualised Volatility	16.1%	14.8%	15.4%	16.4%
Sharpe Ratio	0.50	0.53	0.51	0.38
Semi-Deviation	12.3%	11.4%	11.9%	12.6%
Sortino Ratio	0.66	0.69	0.66	0.49
Maximum Drawdown	-54.4%	-52.1%	-52.6%	-54.9%
Historical Beta	0.97	0.89	0.93	1.00
Excess Return (CAGR)	2.1%	2.1%	2.0%	-
Tracking Error	2.4%	3.3%	2.4%	-
T-stat of Excess Return	3.50	2.15	3.04	-
Information Ratio	0.82	0.51	0.72	-
Active Semi-Deviation	1.5%	2.3%	1.6%	-
Active Sortino Ratio	1.26	0.74	1.06	-
Maximum Active Drawdown	-7.1%	-16.1%	-8.7%	-
Turnover (of aggregate holdings)	40%	36%	36%	3%

The results also confirm that significant diversification benefits can be obtained by combining different return sources into one portfolio. The tracking error of the IR Portfolio is significantly lower (2.4%) than the levels we observed earlier for the various return sources in isolation (3-6%). The standard deviation of the SR Portfolio is lower (14.8%) than that of the most return sources and also the benchmark. Excess returns over the benchmark are statistically different from zero at the 5% confidence level over the studied period for all the portfolios. Analysing active semi-deviations, active Sortino ratios, and maximum active drawdowns, we see that IR Portfolio characteristics are better than those of the equal-weighted portfolio and significantly better than those of the SR Portfolio. Thus, the IR Portfolio was historically the best in delivering consistent excess returns over the studied period (see also Exhibit 8). On the other hand, the SR Portfolio was better with respect to the risk-return trade-off in the absolute space.

---

[9] Portfolios in this exhibit utilize fixed weights discussed in the previous section.

Exhibit 8: Global LC Strategic Portfolios 1 Year Rolling Excess Return (Jul 1997 – Jul 2014)
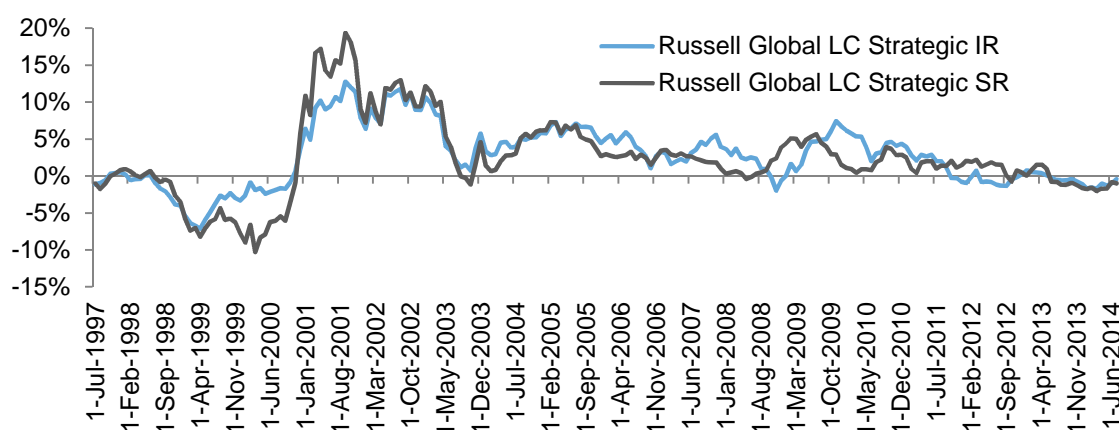
Value, Low Volatility, and Quality are associated with moderate amounts of turnover. Most of the turnover is related to the Momentum factor, but it only has 30% or 20% allocation in our strategic portfolios. Moreover, some trades from the factor portfolios cancel out in a multi-factor allocation. Thus, merging exposures to the return sources in an integrated core portfolio helps to reduce portfolio turnover and minimise transaction costs. The portfolio rebalancing schedule coincides with that of the underlying Factor Exposure portfolios. Rebalancing occurs semi-annually at the end of June and December.

Factor Combination Considerations

There exist a few broad approaches to multi-factor equity investing. In this paper we focus on providing well diversified exposures to the selected strategic return sources for building a strategic equity portfolio as a combination of factor portfolios. In order to be included into a strategic portfolio, a stock has to be included into at least one of the factor portfolios.

Another reasonable approach to a multi-factor equity strategy is to calculate a composite score across multiple factors and then translate this score into a portfolio weight. In this case, it is possible for a stock not to be included into any single factor strategy due to subpar single factor scores but to be included into a composite score multi-factor strategy due to a high combined score[10]. For the most part the returns generated from using a composite score multi-factor strategy and a combination of underlying factor portfolios is similar, however in certain environments a composite score multi-factor strategy may not provide the level of diversification that might be implied from combining multiple factor signals (see Khandani and Lo [2011]), this is contrary to the assertion put forward by some active managers (see Jacobs and Levy [2014]).

Combining factors using either approach is a highly effective strategy, however a composite score strategy can, at extremes, be dominated by a single factor exposure[11]. A multi-factor composite score strategy gives preference to those stocks that display positive characteristics on all factors (not just one) and in environments where there are limited opportunities to display positive characteristics on all, it will favour the characteristic that is the most dominant (i.e. has a greater skew in cross-sectional distribution). Value and Momentum cross-sectional scores are negatively correlated. If we look through history we can find distinct periods where there has been a very large skew in either Value or Momentum cross-sectional scores and it has not necessarily been a good time from a future performance perspective to overweight that factor relative to another one.

Finally, if the goal of the strategy is to build a portfolio through sequential loading on a few factors (e.g., to identify cheap stocks with positive momentum), a third approach can be employed where we use multiple sorts or screens. For example, if we want to build a portfolio of cheap stocks with positive momentum, we can employ a double sorting procedure where we first form quantile portfolios by ranking the stocks according to value

---

[10] In this approach the return comes both from strategic factor exposures and specific security selection.

[11] When evaluating composite score multi-factor equity strategies, investors should pay attention to how the composite score is calculated. It is possible to achieve diversification level similar to the one of combined factor portfolios through the modification of distribution of scores at each point in time. The methodology however will become less intuitive.

characteristics, and then within the top quantile by value characteristic we form quantiles by ranking the stocks according to momentum scores. The final portfolio will be some weighted combination of the stocks in the top quantile after a second sort. This portfolio will be a relatively concentrated portfolio and will have a higher tracking error to the cap-weighted benchmark and a higher turnover level relative to the benchmark.

All of the aforementioned approaches to factor combinations are sensible; however, due to the reasons highlighted above, we believe that asset owners would be best served by strategically allocating to factors in a similar way that they strategically allocate to other return sources, through explicit exposures.

We use a flexible and robust ranking approach to combine several factor portfolios in one portfolio while preserving the important ideas of standard Markowitz mean variance – risk-return trade-off and correlations. In addition, we have researched more than twenty alternative methods of portfolio construction using the same set of four Global factors. Here, we present the results for the seven methods popular among the practitioners. The primary approaches we considered included: equal weight combination (EW), minimum variance or minimum tracking error (MV), maximum Sharpe ratio (MS) or maximum Information ratio (MI), maximum diversification (MD) (Choueifaty and Coignard [2008]), minimum correlation (MC) (Varadi et al. [2012]), naïve risk parity (RP) and equal risk contribution (ERC) (Maillard, Roncalli and Teiletche [2010]).

To allow for out-of-sample performance comparison, we construct portfolios using data from 31 July 1996 to 31 July 2004. We compare the performance of different portfolios over the last ten years from 31 August 2004 to 31 July 2014, i.e., we assume that the strategic portfolios are the same for ten years[12]. Our ranking approach would have selected the same weights using the data from 31 July 1996 to 31 July 2004, i.e., 40% Value/ 30% Momentum/20% Quality/10% Low Volatility for IR Portfolio and 40% Volatility/30% Value/20% Momentum/10% Quality for SR Portfolio[13].

The naïve Risk Parity (RP) portfolio and Equal Risk Contribution (ERC) algorithms provide very similar outcomes in both absolute and active spaces (for our setup with only four factors) and thus we only show the results for the ERC portfolio. The difference between the two is that ERC equalises the risk contribution of each asset to the portfolio and thus uses a correlation matrix while the naïve Risk Parity is just an inverse volatility-weighted portfolio. All the portfolio weights are summarised in Exhibit 9.

Exhibit 9: Portfolio Weighting Summary[14]

	STR. IR	STR. SR	EW	MV (ABS)	MS	MV (ACT)	MI	MD (ABS)	MD (ACT)	MC (ABS)	MC (ACT)	ERC (ABS)	ERC (ACT)
Value	40%	30%	25%	0%	0%	30%	39%	1%	33%	17%	17%	25%	22%
Momentum	30%	20%	25%	0%	8%	7%	24%	37%	18%	31%	22%	22%	20%
Quality	20%	10%	25%	0%	0%	63%	37%	4%	46%	22%	50%	22%	44%
Low Volatility	10%	40%	25%	100%	92%	0%	0%	57%	3%	31%	11%	31%	14%
Total	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**	**100%**

---

[12] Rebalanced to the same weights semiannually.

[13] Although our framework selects the same strategic IR and SR portfolios using the data from 7/31/1996 to 7/31/2004 as in the full sample analysis using the data from 7/31/1996 to 7/31/2014, it is still possible that the results are contingent on the more recent sample period (post 1996) . Unfortunately, our data does not go further back in history and we also could not find external data for the four long only global factor indexes (constructed using consistent methodology) prior to the nineties to extend our out-of-sample analysis. We don't claim that our method is completely agnostic of the sample period but we believe that it reduces the sensitivity to estimation errors in comparison with many alternative portfolio construction methods.

[14] Str. IR – IR portfolio, Str. SR – SR portfolio, EW – equal weight, MV – minimum variance, MS – maximum Sharpe ratio, MI – maximum Information ratio, MD – maximum diversification, MC – minimum correlation, ERC – equal risk contribution. For example, MD (Act) means that the algorithm was applied in the active space and should be compared with IR Portfolio while MD (Abs) means that the algorithm was applied in the absolute space and should be compared with SR Portfolio.

Exhibits 10 and 11 show in-sample (1996-2004) risk-return tradeoffs of the portfolios together with the efficient frontiers in active and absolute spaces, respectively[15].

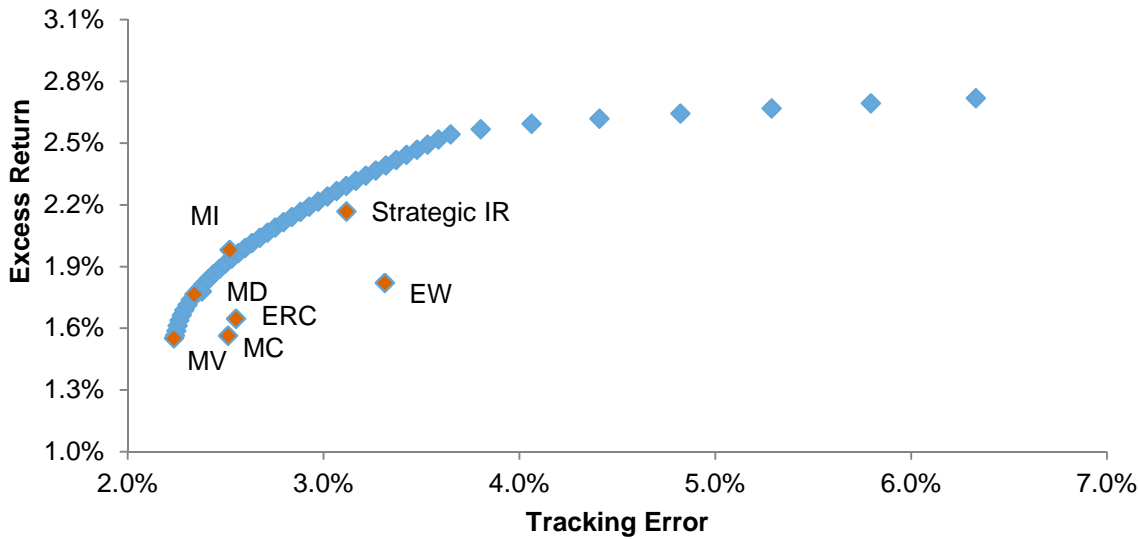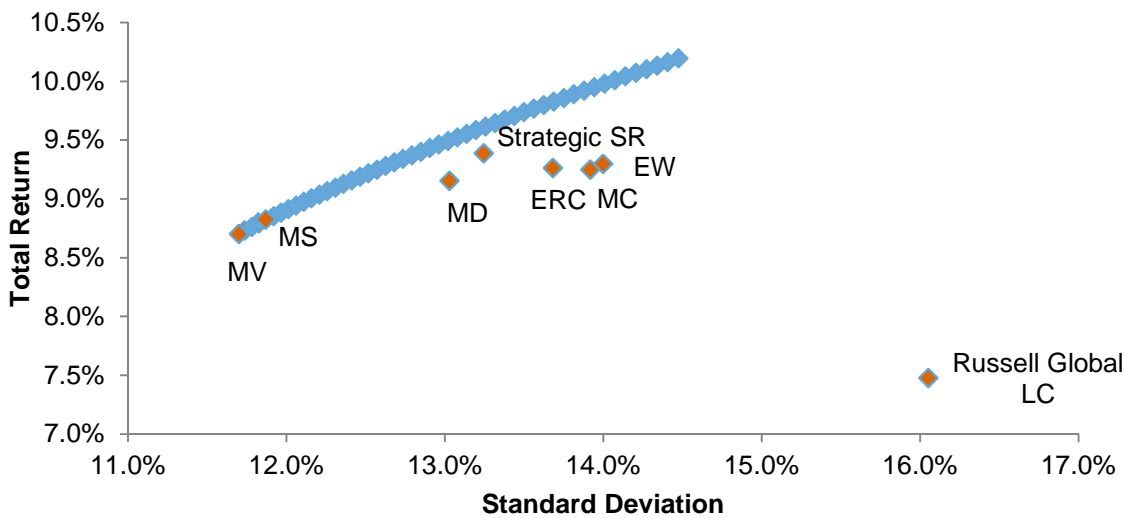**Exhibit 10: Efficient Frontier (Active Space, in-sample)**

Exhibit 11: Efficient Frontier (Absolute Space, in-sample)

---

[15] Some of the portfolios can end up being located slightly higher than the efficient frontier due to the rebalancing effect (all the portfolios that we compare are rebalanced semi-annually).

Exhibits 12 and 13 show the out-of-sample (2004-2014) risk-return tradeoffs of the portfolios, together with the out-of-sample frontiers in active and absolute spaces, respectively. The IR Portfolio and the SR Portfolio stay close to the corresponding out-of-sample efficient frontiers.

Exhibit 12: Efficient Frontier (Active Space, out-of-sample)
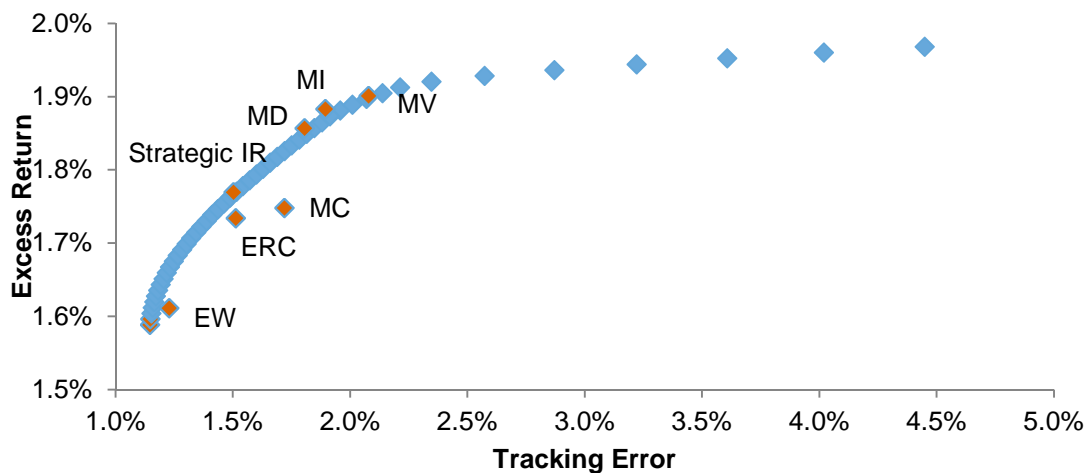
Exhibit 13: Efficient Frontier (Absolute Space, out-of-sample)
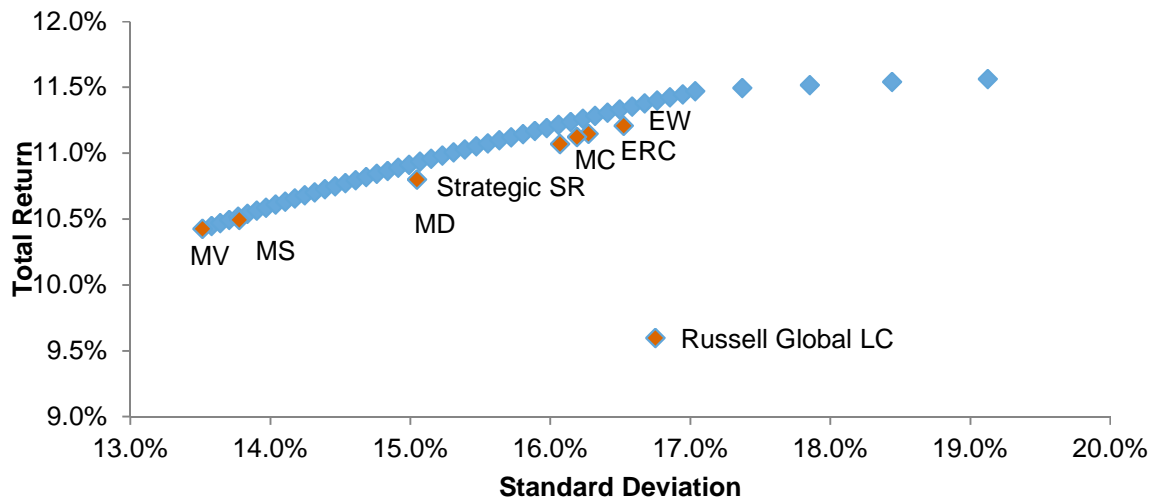
Our main goal is to compare alternative weighting approaches in a strategic allocation framework (i.e., choosing the weights using the data from one period and then comparing the out-of-sample performance of fixed weights portfolios over the second period). We believe that our ranking approach works well in the strategic allocation framework. However, it is possible that the alternative approaches work better if the weights are constantly updated and are sensitive to recent observations. To investigate this, we also allow for time-varying weights in the alternative schemes by performing a 36-month window estimation of parameters. For example, for the naïve Risk Parity portfolio, where the factor weight is inversely proportional to volatility, volatility estimate is changing semi-annually and is estimated using the recent 36 month of returns data. The results are presented in Exhibit 14 (the portfolios are rebalanced semi-annually). After studying these results, we still do not see any substantial difference when using alternative weighting schemes to combine factors especially given that this dynamic weighting generates higher turnover[16] .

---

[16] Some of the approaches might be preferred when leverage is allowed. Also these alternative approaches might still work better for the portfolios with large number of assets (e.g., at the security level) – this analysis is out of the scope of this paper. The weights for portfolios other than EW, Str. IR and Str. SR, are evolving over time using a rolling window of history to estimate parameters.

Exhibit 14: Alternative weighting schemes (Global LC, Jan 2000 – Jul 2014)[17]

ACTIVE SPACE	STR. IR	EW	MV	MI	MD	MC	ERC
Annualised Return (CAGR)	7.4%	7.3%	7.2%	5.8%	7.3%	7.0%	7.1%
Annualised Volatility	16.5%	15.7%	16.4%	17.2%	16.2%	16.1%	16.1%
Sharpe Ratio	0.41	0.41	0.40	0.31	0.41	0.39	0.40
Semi-Deviation	12.4%	11.8%	12.2%	13.2%	12.1%	12.0%	12.0%
Sortino Ratio	0.54	0.55	0.53	0.40	0.54	0.51	0.52
Maximum Drawdown	-54.4%	-52.6%	-51.2%	-52.9%	-51.1%	-52.0%	-51.7%
Historical Beta	0.98	0.93	0.97	1.01	0.96	0.96	0.96
Excess Return (CAGR)	3.0%	3.0%	2.9%	1.4%	2.9%	2.6%	2.7%
Tracking Error	2.4%	2.4%	2.2%	3.9%	2.4%	2.0%	2.1%
T-stat of Excess Return	4.40	4.10	4.57	1.39	4.23	4.36	4.47
Information Ratio	1.18	1.10	1.23	0.37	1.14	1.17	1.20
Active Semi-Deviation	1.5%	1.6%	1.4%	3.0%	1.5%	1.3%	1.3%
Active Sortino Ratio	1.84	1.65	1.99	0.49	1.79	1.82	1.85
Maximum Active Drawdown	-4.7%	-5.0%	-3.3%	-17.0%	-3.5%	-4.0%	-3.8%
ABSOLUTE SPACE	**STR. IR**	**EW**	**MV**	**MI**	**MD**	**MC**	**ERC**
Annualised Return (CAGR)	7.8%	7.3%	8.0%	6.7%	7.0%	7.3%	7.5%
Annualised Volatility	15.2%	15.7%	13.0%	17.8%	15.3%	15.4%	15.5%
Sharpe Ratio	0.45	0.41	0.52	0.35	0.40	0.42	0.43
Semi-Deviation	11.4%	11.8%	9.9%	13.0%	11.5%	11.6%	11.6%
Sortino Ratio	0.60	0.55	0.67	0.48	0.53	0.55	0.56
Maximum Drawdown	-52.1%	-52.6%	-46.6%	-51.6%	-50.5%	-51.4%	-52.1%
Historical Beta	0.90	0.93	0.74	1.02	0.90	0.91	0.92
Excess Return (CAGR)	3.5%	3.0%	3.6%	2.3%	2.7%	2.9%	3.1%
Tracking Error	3.2%	2.4%	6.0%	5.4%	3.8%	2.8%	2.6%
T-stat of Excess Return	3.50	4.10	1.79	1.62	2.31	3.43	3.88
Information Ratio	0.94	1.10	0.48	0.44	0.62	0.92	1.04
Active Semi-Deviation	2.1%	1.6%	4.1%	3.5%	2.6%	1.8%	1.8%
Active Sortino Ratio	1.43	1.65	0.69	0.67	0.89	1.39	1.57
Maximum Active Drawdown	-5.5%	-5.0%	-9.8%	-17.0%	-6.9%	-5.6%	-5.2%

[17]Str. IR – IR portfolio, Str. SR – SR portfolio, EW – equal weight, MV – minimum variance, MI – maximum Information ratio, MS – maximum Sharpe ratio, MD – maximum diversification, MC – minimum correlation, ERC – equal risk contribution.

Conclusions

In this paper, we present two strategic equity portfolios that combine four well-documented return sources: Value, Momentum, Quality, and Low Volatility to meet two distinct objectives. Combining several return sources in one portfolio exploits diversification benefits of the factors and potentially delivers more consistent performance. Investors may use multi-factor allocations to target an absolute (Sharpe ratio) or a relative (Information ratio) risk-return objective.

We propose a robust and flexible framework that uses the principles of modern portfolio theory, and reduces the sensitivity to estimation errors. Our framework moves away from single-point estimates and captures the high-level relationships of return sources through a ranking approach. These high-level relationships have shown to hold through time as well as allow for consistent exposures through time. When determining portfolio allocations to various return sources we encourage investors to focus on high-level relationships as opposed to singe-point estimates.

References

Amenc N., Goltz F., Lodh A. and Martellini L. "Towards Smart Equity Factor Indices: Harvesting Risk Premia without Taking Unrewarded Risks", The Journal of Portfolio Management 40, No. 4 (2014)

Barber, J., Bennett S. and Paris M., "Russell High Efficiency Factor Index Series: Providing investors with efficient exposures to return drivers", Russell Research (2014)

Bender J., Brandhorst E. and Wang T., "The Latest Wave in Advanced Beta: Combining Value, Low Volatility and Quality", The Journal of Index Investing 5, No.1 (2014)

Choueifaty, Y. and Coignard, Y. "Toward Maximum Diversification", Journal of Portfolio Management 34, No.4 (2008), pp. 40-51

Greene, J., Hand, J.R.M., and Zhang, X.F. "The Supraview of Return Predictive Signals." Review of Accounting Studies 18 (2013), pp. 692-730

Jacobs, I. B. and Levy, K.N. "Investing in a Multidimensional Market." Financial Analysts Journal 70, No. 6, (November-December 2014)

Khandani A.E., and Lo A.W. "What happened to the Quants in August 2007? Evidence from factors and transaction data" Journal of Financial Markets, Volume 14, Issue 1, (February 2011), pp 1–46

Litterman, et al. "Modern Investment Management: An Equilibrium Approach by Bob Litterman and the Quantitative Research Group." Goldman Sachs Asset Management (2003), Chapter 7

Lo A.W. "Where Do Alphas Come From?: A Measure of the Value of Active Investment Management." Journal of Investment Management , 3rd Quarter (2008)

Maslov, D., and Rytchkov, O. "Robustness and Monotonicity of Asset Pricing Anomalies", Working paper, University of Texas at Austin , 2013

Michaud. R.O. "The Markowitz Optimisation Enigma: Is 'Optimised' Optimal?" Financial Analysts Journal 45, No.1, (January- February 1989), pp. 31-42

Maillard, S., Roncalli, T., and Teïletche, J. "The Properties of Equally Weighted Risk Contribution Portfolios", Journal of Portfolio Management, 36, No.4 (2010), pp. 60-70

Varadi, D., Kapler M., Bee H. and Rittenhouse C. "The Minimum Correlation Algorithm: A Practical Diversification Tool", Working paper

Appendix

Alternative weighting schemes details:

Equal weight (EW) – A type of weighting that gives the same weight to each asset in a portfolio.

Minimum variance (MV) – A type of weighting that solves the following optimisation problem:

$$\min_{w} w^T \Sigma w$$

subject to $\sum_i w_i = 1, 0 \leq w_i \leq 1$, where Σ is a covariance matrix of absolute or excess returns.

Maximum Sharpe ratio (MS) (or **Maximum Information ratio (MI)** in the active space) – A type of weighting that solves the following optimisation problem:

$$\max_{w} w^T \mu / sqrt(w^T \Sigma w)$$

subject to $\sum_i w_i = 1, 0 \leq w_i \leq 1$, where μ is an excess return over a risk free rate or excess return over a benchmark in active space, Σ is a covariance matrix of absolute or excess returns.

Maximum diversification (MD) – A type of weighting that solves the following optimisation problem (Choueifaty and Coignard [2008]):

$$\min_{w} w^T P w$$

subject to $\sum_i w_i = 1, 0 \leq w_i \leq 1$, where P is a correlation matrix of absolute or excess returns

$w_i^* = \frac{w_i / \sigma_i}{\sum_i w_i / \sigma_i}$, where σ is a vector of volatilities of absolute or excess returns of assets in a portfolio.

Minimum correlation (MC) – A type of weighting that is obtained using the following steps (Varadi et al. [2012]):

- Compute mean(μ_ρ) and standard deviation(σ_ρ) of elements of a correlation matrix P (of absolute or excess returns).

- Create adjusted correlation matrix, P_A, by transforming each element of the original correlation matrix from -1 to +1 space to 1 to 0 space using 1 – Normal Inverse Transformation(μ_ρ, σ_ρ) .

- Compute average value for each row of $P_A - w_i$.

- $w_i^* = \frac{RANK(w_i)}{\sum_i RANK(w_i)}$

- $w_i^{**} = \frac{w_i^* * P_A}{\sum_i w_i^* P_A}$

- $w_i^{***} = \frac{w_i^{**} / \sigma_i}{\sum_i w_i^{**} / \sigma_i}$, where σ is a vector of volatilities of absolute or excess returns of assets in a portfolio.

Risk parity (naïve) (RP) – A type of weighting where weights are proportional to the inverse of volatilities of absolute or excess returns.

Equal risk contribution (ERC) – A type of weighting that solves the following optimisation problem (Maillard, Roncalli and Teiletche [2010]):

$$\min_{w} \sum_i \sum_j \left(w_i (\Sigma w)_i - w_j (\Sigma w)_j \right)^2$$

subject to $\sum_i w_i = 1, 0 \leq w_i \leq 1$, where Σ is a covariance matrix of absolute or excess returns.

ABOUT RUSSELL INVESTMENTS

Russell Investments provides strategic advice, world-class implementation, state-of-the-art performance benchmarks and a range of institutional-quality investment products, serving clients in more than 35 countries. Russell provides access to some of the world's best money managers. It helps investors put this access to work in defined benefit, defined contribution, public retirement plans, endowments and foundations and in the life savings of individual investors.

FOR MORE INFORMATION:

Call Russell at **+44 (0) 20 7024 6000** or
visit **www.russell.com**